# NAG Toolbox for MATLAB

# f08av

## 1 Purpose

f08av computes the $LQ$ factorization of a complex $m$ by $n$ matrix.

## 2 Syntax

```
[a, tau, info] = f08av(a, 'm', m, 'n', n)
```

## 3 Description

f08av forms the $LQ$ factorization of an arbitrary rectangular complex $m$ by $n$ matrix. No pivoting is performed.

If $m \leq n$, the factorization is given by:

$$A = (L \quad 0)Q$$

where $L$ is an $m$ by $m$ lower triangular matrix (with real diagonal elements) and $Q$ is an $n$ by $n$ unitary matrix. It is sometimes more convenient to write the factorization as

$$A = (L \quad 0)\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$$

which reduces to

$$A = LQ_1,$$

where $Q_1$ consists of the first $m$ rows of $Q$, and $Q_2$ the remaining $n - m$ rows.

If $m > n$, $L$ is trapezoidal, and the factorization can be written

$$A = \begin{pmatrix} L_1 \\ L_2 \end{pmatrix} Q$$

where $L_1$ is lower triangular and $L_2$ is rectangular.

The $LQ$ factorization of $A$ is essentially the same as the $QR$ factorization of $A^{\mathrm{H}}$, since

$$A = (L \quad 0)Q \Leftrightarrow A^{\mathrm{H}} = Q^{\mathrm{H}}\begin{pmatrix} L^{\mathrm{H}} \\ 0 \end{pmatrix}.$$

The matrix $Q$ is not formed explicitly but is represented as a product of $\min(m, n)$ elementary reflectors (see the F08 Chapter Introduction for details). Functions are provided to work with $Q$ in this representation (see Section 8).

Note also that for any $k < m$, the information returned in the first $k$ rows of the array **a** represents an $LQ$ factorization of the first $k$ rows of the original matrix $A$.

## 4 References

None.

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **a(lda,$*$) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The $m$ by $n$ matrix $A$.

### 5.2 Optional Input Parameters

1: **m – int32 scalar**

*Default*: The first dimension of the array **a**.

$m$, the number of rows of the matrix $A$.

*Constraint*: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

*Default*: The second dimension of the array **a**.

$n$, the number of columns of the matrix $A$.

*Constraint*: $\mathbf{n} \geq 0$.

### 5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork

### 5.4 Output Parameters

1: **a(lda,$*$) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If $m \leq n$, the elements above the diagonal are overwritten by details of the unitary matrix $Q$ and the lower triangle contains the corresponding elements of the $m$ by $m$ lower triangular matrix $L$.

If $m > n$, the strictly upper triangular part contains details of the unitary matrix $Q$ and the remaining elements are overwritten by the corresponding elements of the $m$ by $n$ lower trapezoidal matrix $L$.

The diagonal elements of $L$ are real.

2: **tau($*$) – complex array**

**Note**: the dimension of the array **tau** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

further details of the unitary matrix $Q$.

3: **info – int32 scalar**

**info** $= 0$ unless the function detects an error (see Section 6).

## 6 Error Indicators and Warnings

Errors or warnings detected by the function:

**info** $= -i$

If **info** $= -i$, parameter $i$ had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **a**, 4: **lda**, 5: **tau**, 6: **work**, 7: **lwork**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

## 7 Accuracy

The computed factorization is the exact factorization of a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and $\epsilon$ is the ***machine precision***.

## 8 Further Comments

The total number of real floating-point operations is approximately $\frac{8}{3}m^2(3n - m)$ if $m \leq n$ or $\frac{8}{3}n^2(3m - n)$ if $m > n$.

To form the unitary matrix $Q$ f08av may be followed by a call to f08aw:

```
[a, info] = f08aw(a(1:n,:), tau);
```

but note that the first dimension of the array **a**, specified by the parameter **lda**, must be at least **n**, which may be larger than was required by f08av.

When $m \leq n$, it is often only the first $m$ rows of $Q$ that are required, and they may be formed by the call:

```
[a, info] = f08aw(a, tau, 'k', m);
```

To apply $Q$ to an arbitrary complex rectangular matrix $C$, f08av may be followed by a call to f08ax. For example,

```
[c, info] = f08ax('Left','Conjugate Transpose', a(:,1:p), tau, c);
```

forms the matrix product $C = Q^{\mathrm{H}}C$, where $C$ is $m$ by $p$.

The real analogue of this function is f08ah.

## 9 Example

```
a = [complex(0.28, -0.36), complex(0.5, -0.86), complex(-0.77, -0.48),
complex(1.58, +0.66);
     complex(-0.5, -1.1), complex(-1.21, +0.76), complex(-0.32, -0.24),
complex(-0.27, -1.15);
          complex(0.36, -0.51), complex(-0.07000000000000001, +1.33),
complex(-0.75, +0.47), complex(-0.08, +1.01)];
[aOut, tau, info] = f08av(a)
```

```
aOut =
  -2.2255                 0.2438 - 0.3082i  -0.2741 - 0.2310i   0.5808 +
0.3469i
   0.8208 + 1.2385i   1.6881                -0.1936 + 0.5430i   0.2789 -
0.2203i
   0.0010 - 0.6822i   0.7748 - 0.6155i  -1.5903               -0.1268 +
0.1110i
tau =
   1.1258 + 0.1618i
   1.0991 + 0.5469i
   1.1329 - 0.9591i
info =
           0
```